

LOGARITHM & QUADRATIC EQUATION

$$Q \log_{10} \left(\frac{a+b + \sqrt{(a+b)^2 - 4(a+b)}}{2} \right) + \log_{10} \left(\frac{a+b - \sqrt{(a+b)^2 - 4(a+b)}}{2} \right) = \log_{10}(x)$$

find x?

$$\log_{10} \left\{ \left(\frac{a+b + \sqrt{(a+b)^2 - 4(a+b)}}{2} \right) \left(\frac{a+b - \sqrt{(a+b)^2 - 4(a+b)}}{2} \right) \right\}$$

$\log A + \log B$
 $\frac{A+B}{2}$

$$\log_{10} \left\{ \frac{a^2 + b^2 - ((a+b)^2 - 4(a+b))}{4} \right\} = \log_{10} x$$

$$\log_{10}(a+b) = \log_{10} x \Rightarrow \boxed{x = a+b}$$

LOGARITHM & QUADRATIC EQUATION

Fundamental Identity.

$$1) a^{\log_a N} = N$$

Proof $N = a^x$

$$\text{LHS } a^{\log_a a^x} = a^{x \log_a a}$$

$$= a^{x \times 1} = a^x = N = \text{RHS}$$

$$\text{Ex: } 2^{\log_2 5} = 5$$

$$2) e^{\ln x} = e^{\log_e x} = x$$

$$3) \left(\frac{1}{2}\right)^{\log_2 5} = 2^{-1 \times \log_2 5} = 2^{\log_2 5^{-1}} = 5^{-1} = \frac{1}{5}$$

LOGARITHM & QUADRATIC EQUATION

$$(4) \left(\frac{1}{9}\right)^{\log_3 7}$$

$$3^{-2 \log_3 7} = 7^{-2} = \frac{1}{49}$$

$$(5) 2^{3 \log_2 3} = 2^{\log_2 3^3}$$

$$= 3^3 = 27$$

$$Q \log_6 9 - \log_5 27 + \log_8 x = \log_{14} x - \log_6 4$$

$$Q \ 2^{\log_2 x^2} - 3x - 4 = 0$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$$x = 4, -1$$

LOGARITHM & QUADRATIC EQUATION

① If $K^{\log_2 5} = 16$ then value of

$$K^{(\log_2 5)^2} = ?$$

$$K^{(\log_2 5)^2} = K^{\log_2 5 \times \log_2 5} = (K^{\log_2 5})^{\log_2 5}$$

$$= (16)^{\log_2 5} = 2^{4 \log_2 5}$$

$$= 2^{\log_2 5^4} = 5^4 = 625$$

$$a^{m \times n} = (a^m)^n$$

Q $a^{\log_3 7} = 27$, $b^{\log_7 11} = 49$

Value of $a^{(\log_3 7)^2} + b^{(\log_7 11)^2} = ?$

$$(a^{\log_3 7})^{\log_3 7} + (b^{\log_7 11})^{\log_7 11}$$

$$(27)^{\log_3 7} + (49)^{\log_7 11}$$

$$3^{3 \log_3 7} + 7^{2 \log_7 11} = 7^3 + 11^2$$

$$343 + 121 = 464$$

LOGARITHM & QUADRATIC EQUATION

(2) Converting Exp. value $\Rightarrow$

$$\boxed{\ln x = \log_e x}$$

exp value

$$a^x = e^{x \ln a}$$

$$2^4 = e^{4 \ln 2}$$

$$3^2 = e^{2 \ln 3}$$

$$Q \quad 2^{\ln z} = e^{\ln z \cdot \ln 2}$$

$$\neq e^{\ln(z+2)}$$

$$Q \quad \text{Find value of } 2^{\log_2 e^{\ln 5}} = ?$$

$$= e^{\log_2 e^5} = 5$$

LOGARITHM & QUADRATIC EQUATION

Q Find x

$$\cancel{2}^{\log_2} \cancel{5}^{\log_5} \cancel{7}^{\log_7} \cancel{3}^{\log_3} (8x-3) = 13$$

$$8x-3=13$$

$$x = \frac{16}{8} = 2$$

(3) B.C.T.

Base change Theorem.

It states that the quotient of log of 2 No. is Independent of their common Base.

$$\frac{\log_a M}{\log_a N} = \log_n M$$

LOGARITHM & QUADRATIC EQUATION

$\log_N M \rightarrow$ Base a Required

$$= \frac{\log_a M}{\log_a N} = \frac{\log_b M}{\log_b N} = \frac{\log_e M}{\log_e N}$$

Normally we Use BCT when 2 log are Multiplied or divided.

Q value of

$$\log_b a \cdot \log_a b = ?$$

$$\frac{\cancel{\log_e a}}{\cancel{\log_e b}} \times \frac{\cancel{\log_e b}}{\cancel{\log_e a}} = 1$$

LOGARITHM & QUADRATIC EQUATION

4) Results

$$A) \frac{1}{\log_A B} = \log_B A$$

$$B) \log_{N^\alpha} M^B = \frac{B}{\alpha} \log_N M$$

$$\text{BCT} \Rightarrow \frac{\log_e M^B}{\log_e N^\alpha} = \frac{B \log_e M}{\alpha \log_e N} = \frac{B}{\alpha} \log_N M$$

$$Q \frac{1}{\log_{xy}(xyz)} + \frac{1}{\log_{yz}(xyz)} + \frac{1}{\log_{zx}(xyz)} = 2$$

$$\log_{xyz} xy + \log_{xyz} yz + \log_{xyz} zx$$

$$\log_{xyz} (xy \cdot yz \cdot zx)$$

$$= \log_{xyz} (xyz)^2$$

$$= 2 \log_{xyz} xyz = 2$$

LOGARITHM & QUADRATIC EQUATION

Q $\log_{\boxed{2}} 3 \cdot \log_{\boxed{3}} 4 \cdot \log_{\boxed{4}} 5 \cdot \log_5 6 \dots \log_n (n+1) = 10$ find n ?

log Multi $\rightarrow$ B.C.T.

$$\frac{\cancel{\log 3}}{\log 2} \times \frac{\cancel{\log 4}}{\cancel{\log 3}} \times \frac{\cancel{\log 5}}{\cancel{\log 4}} \times \frac{\log 6}{\log 5} \times \dots \times \frac{\log(n+1)}{\cancel{\log n}} = 10$$

$$\frac{\log(n+1)}{\cancel{\log 2}} = 10 \Rightarrow \cancel{\log}_2(n+1) = 10$$

$$n+1 = 2^{10} = 1024$$

$$n = 1023$$

LOGARITHM & QUADRATIC EQUATION

Q Find N if $N = \sum_{r=2}^{2023} \frac{1}{\log_r P}$ Where $P = (2 \times 3 \times 4 \times 5 \times \dots \times 2023)$

$$\frac{1}{\log_2 P} + \frac{1}{\log_3 P} + \frac{1}{\log_4 P} + \frac{1}{\log_5 P} + \dots + \frac{1}{\log_{2023} P} = N$$

$$\log_P 2 + \log_P 3 + \log_P 4 + \log_P 5 + \dots + \log_P 2023 = N$$

$$\log_P (2 \times 3 \times \dots \times 2023) = N$$

$$\log_P P = N = 1$$

$$\frac{1}{\log_A B} = \log_B A$$

LOGARITHM & QUADRATIC EQUATION

$$Q \log_3 2 = \log_{27} 8 \quad [T/F]$$

$$\log_3 2 = \log_{3^3} 2^3$$

$$= \frac{3}{3} \log_3 2$$

$$= \log_3 2$$

$$Q \log_{81} 73 = \log_{\sqrt{81}} \sqrt{73} \quad [T/F]$$

$$= \frac{1}{2} \log_{81} 73$$

$$Q \frac{1}{\log_3 2} + \frac{2}{\log_9 4} - \frac{3}{\log_{27} 8} = ?$$

$$\left\{ \frac{1}{\log_3 2} + \frac{2}{\log_3 2} \right\} - \frac{3}{\log_3 2}$$

$$\frac{3}{\log_3 2} - \frac{3}{\log_3 2} = 0$$

LOGARITHM & QUADRATIC EQUATION

$$\log_{2^3} x^3 = \frac{3}{3} \log_2 x$$

Q $\log_{\sqrt{2}} \sqrt{x} + \log_2 x + \log_4 x^2 + \log_8 x^3 + \log_{16} x^4 = 40$ find x ?

$$\frac{1}{\frac{1}{2}} \log_2 x + \log_2 x + \frac{x}{2} \log_2 x + \frac{3}{8} \log_2 x + \frac{4}{4} \log_2 x = 40$$

$$8 \log_2 x = 40$$

$$x = 2^8 = 256$$

$$\log_3 x = 4$$

$$x = 3^4$$

$$\log_N x = A$$

$$x = N^A$$

$$\log_2 x = 10$$

$$x = 2^{10}$$

LOGARITHM & QUADRATIC EQUATION

Q Value of $\frac{2}{\log_4 (2000)^6} + \frac{3}{\log_5 (2000)^6} = ?$

$$2 \log_{(2000)^6} 4 + 3 \log_{(2000)^6} 5$$

$$\log_{(2000)^6} 4^2 + \log_{(2000)^6} 5^3$$

$$\log_{(2000)^6} 16 \times 125 = \frac{1}{6} \log_{(2000)^6} 2000 = \frac{1}{6}$$

log → Algebraic
Strength
Improve

LOGARITHM & QUADRATIC EQUATION

5) Result

$$a^{\log_b c} = b^{\log_c a}$$

$$\ln x = \log_e x$$

$$a^{\frac{\log_a b}{\log_a c}} = a^{\log_a b \times \frac{1}{\log_a c}} = \left(a^{\log_a b} \right)^{\frac{1}{\log_a c}} = (b)^{\log_c a}$$

$$a^{m \times n} = (a^m)^n$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$Q \int 2^{\ln x} dx$$

$$\Rightarrow \int x^{\ln 2} dx$$

$$\frac{x^{\ln 2 + 1}}{\ln 2 + 1} + C$$

LOGARITHM & QUADRATIC EQUATION

Q $7^{\log_3 5} - 3^{\log_7 5} - 5^{\log_3 7} + 5^{\log_7 3} + 5$

~~$7^{\log_3 5} - 3^{\log_7 5}$~~ ~~$- 7^{\log_3 5} + 3^{\log_7 5}$~~ + 5

DROPPER

= 5

$$2^{\log_3 x} = 2^2$$

$$\log_3 x = 2$$

$$x = 3^2 = 9$$

Q Find x if

$$2^{\log_3 x} + 8 = 3 \cdot x^{\log_9 4 \rightarrow 2^2}$$

$$2^{\log_3 x} + 8 = 3 \cdot x^{\log_3 2}$$

$$\boxed{2^{\log_3 x}} + 8 = 3 \cdot \boxed{2^{\log_3 x}}$$

$$t + 8 = 3t$$

$$t = 4$$

LOGARITHM & QUADRATIC EQUATION

$$Q \quad \frac{\log_3 12}{\log_3 36} - \frac{\log_3 4}{\log_3 108} = 2 \quad \log_3 2 = t$$

$$\text{LHS} \quad \log_3 12 \times \log_3 36 - \log_3 4 \times \log_3 108$$

$$\log_3 (2^2 \times 3) \times \log_3 (2^2 \times 3^2) - \log_3 2^2 \times \log_3 (2^2 \times 3^3)$$

$$(\log_3 2^2 + \log_3 3) \times (\log_3 2^2 + \log_3 3^2) - \log_3 2^2 \times (\log_3 2^2 + \log_3 3^3)$$

$$(2 \log_3 2 + 1) \times (2 \log_3 2 + 2 \times 1) - 2 \log_3 2 \times (2 \log_3 2 + 3 \times 1)$$

$$(2t+1)(2t+2) - 2t(2t+3)$$

$$(\cancel{4t^2} + 6t + 2) - \cancel{4t^2} - 6t$$

$$= 2 = \text{RHS.}$$

LOGARITHM & QUADRATIC EQUATION

$$Q \quad x^{\frac{\ln y - \ln z}{\ln x}} \cdot y^{\frac{\ln z - \ln x}{\ln y}} \cdot z^{\frac{\ln x - \ln y}{\ln z}}$$

Converting in exp form

$$2^4 = e^{4 \ln 2}$$

$$e^{(\ln y - \ln z) \frac{\ln x}{\ln x}} \cdot e^{(\ln z - \ln x) \frac{\ln y}{\ln y}} \cdot e^{(\ln x - \ln y) \frac{\ln z}{\ln z}}$$

30 Chapter

Average + Around

15 Chapter

$$e^{\cancel{\ln x} \cancel{\ln y - \ln z} \cancel{\ln z} + \cancel{\ln y} \cancel{\ln z - \ln x} \cancel{\ln y} + \cancel{\ln x} \cancel{\ln z - \ln y} \cancel{\ln z}} = e^0 = 1$$

LOGARITHM & QUADRATIC EQUATION

$$\ln x + \ln y = -\ln z$$

Q Find value of $x^{\left(\frac{1}{\ln y} + \frac{1}{\ln z}\right)} \cdot y^{\frac{1}{\ln z} + \frac{1}{\ln x}} \cdot z^{\frac{1}{\ln x} + \frac{1}{\ln y}}$?

5-7

exp form & convert

$$\text{if } \boxed{\ln x + \ln y + \ln z = 0}$$

$$e^{-\frac{\ln z}{\ln z} + -\frac{\ln x}{\ln x} + -\frac{\ln y}{\ln y}} \cdot e^{\left(\frac{1}{\ln y} + \frac{1}{\ln z}\right) \ln x} \cdot e^{\left(\frac{1}{\ln z} + \frac{1}{\ln x}\right) \ln y} \cdot e^{\left(\frac{1}{\ln x} + \frac{1}{\ln y}\right) \ln z}$$

$$e^{-3} = \frac{1}{e^3} \quad \left| \quad e^{\frac{\ln x}{\ln y} + \left(\frac{\ln x}{\ln z} + \frac{\ln y}{\ln z}\right) + \frac{\ln y}{\ln x} + \frac{\ln z}{\ln x} + \frac{\ln z}{\ln y}} \right.$$

$$\left. e^{\frac{(\ln x + \ln y)}{\ln z} + \frac{(\ln y + \ln z)}{\ln x} + \frac{(\ln x + \ln z)}{\ln y}} \right.$$

LOGARITHM & QUADRATIC EQUATION

Q Find value of x

$$\log_3 x \cdot \log_4 x \cdot \log_5 x = \log_3 x \cdot \log_4 x + \log_4 x \cdot \log_5 x + \log_5 x \cdot \log_3 x$$

$$\frac{\log x}{\log 3} \times \frac{\log x}{\log 4} \times \frac{\log x}{\log 5} = \frac{\log x}{\log 3} \times \frac{\log x}{\log 4} + \frac{\log x}{\log 4} \times \frac{\log x}{\log 5} + \frac{\log x}{\log 5} \times \frac{\log x}{\log 3}$$

$$\frac{(\log x)^3}{\log 3 \log 4 \log 5} = (\log x)^2 \left\{ \frac{1}{\log 3 \log 4} + \frac{1}{\log 4 \log 5} + \frac{1}{\log 5 \log 3} \right\}$$

$$\frac{(\log x)^3}{\cancel{\log 3} \times \cancel{\log 4} \times \cancel{\log 5}} = (\log x)^2 \left\{ \frac{\log 5 + \log 3 + \log 4}{\cancel{\log 3} \times \cancel{\log 4} \times \cancel{\log 5}} \right\}$$

$$(\log x)^3 = (\log x)^2 \times (\log 60)$$

$$(\log x)^2 \{ \log x - \log 60 \} = 0$$

$$(\log x)^2 = 0 \text{ OR } \log x = \log 60$$

$$\cancel{\log} x = 0$$

$$x = e^0 = 1$$

$$\boxed{x = 60}$$

LOGARITHM & QUADRATIC EQUATION

Trickotomy.

Q $A = \log_{\boxed{.5}} 25$ & $B = \log_2 3$

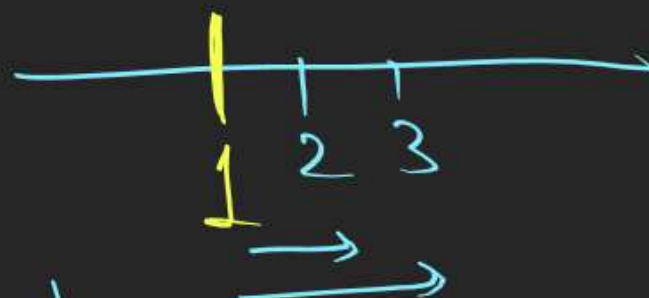
Which one is grt.



1 to Idhr / Udhr.

$\therefore A = -ve$

$B > A$



1 to Same Side

$B = +ve$

Q $A = \log_3 \pi$, $B = \log_\pi 3$
 (+) NO I h? (+)



Is k Exponent, Base & asi
 that will be hr. than 1

$A > 1$ | $B < 1$
 $A > B$